Hall Ticket Number:

Code No. : 12002 O2

VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD
B.E. I Year II-Semester Backlog Examinations, May-2017
Mathematics-II

Time: 3 hours   Max. Marks: 50

Note: Answer ALL questions in Part-A and any FIVE from Part-B

Part-A (15 Marks)

1. Find the directional derivative of $f(x, y, z) = xy^2 + 4xyz + z^2$ at the point $(1, 2, 3)$ in the direction $(i - j + 2k)$.   [1]

2. What is Integrating factor?   [1]

3. Solve the differential equation $y''' - y'' - 4y' + 4y = 0$.   [1]

4. Express $3x^2 + 5x - 6$ in terms of Legender polynomial.   [1]

5. Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.   [1]

6. Evaluate $\int_c (x^2 - y^2) ds$, c is the closed curve $x = 3\cos t, y = 3\sin t, 0 \leq t \leq 2\pi$.   [2]

7. Find orthogonal trajectory of $y = x + ce^{-x}$.   [2]

8. Write down particular integral of the differential equation $y''' - 2y'' - 5y' + 6y = 18e^x$.   [2]

9. Define ordinary and singular points of an equation.   [2]

10. Define Beta and Gamma function.   [2]

Part-B (5 × 7 = 35Marks)

11. a) If $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$, show that the $\text{div}\left(\frac{\bar{r}}{r^3}\right) = 0$ [3]

    b) Apply Green's theorem to evaluate $\int_c [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where c is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.   [4]

12. a) Solve the differential equation $y' + 4xy + xy^3 = 0$.   [3]

    b) Show that the one parameter family of curves $y^2 = 4c(x + c)$ are self orthogonal.   [4]

13. a) Find the general solution of the equation $y'' + 16y = 32\sec 2x$, using the method of variation of parameters.   [4]

    b) Find the solution of the differential equation $x^2y'' + 2xy' - 2y = 0$.   [3]

14. a) Find the power series solution about $x = 2$, of the initial value problem $4y'' - 4y' + y = 0, y(2) = 0, y'(2) = \frac{1}{e}$.   [3]

    b) Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$ [4]

15. a) Prove that $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ [4]

   b) Show that $J_{\frac{5}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}\left[\dfrac{1}{x^2}(3-x^2)\sin x - \dfrac{3}{x}\cos x\right]$ [3]

16. a) Evaluate the line integral of $\mathbf{v} = x^2\mathbf{i} - 2y\mathbf{j} + z^2\mathbf{k}$ over the straight line path from $(-1,2,3)$ [3] to $(2,3,5)$

   b) Obtain the general solution and the singular solution of the non-linear equation [4] $y = xy' + (y')^2$

17. Answer any **two** of the following: [7]

   a) Find the general solution of the differential equation $x^3 y''' + 2x^2 y'' = 0$

   b) Using Rodrigue's formula, show that $\displaystyle\int_{-1}^{1} x^m P_n(x)\,dx = 0, m < n$.

   c) Show that $J_{\frac{1}{2}}^2(x) + J_{\frac{-1}{2}}^2(x) = \dfrac{2}{\pi x}$

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